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## THE DYNAMIC PATTERN APPROACH TO COORDINATED BEHAVIOR: A TUTORIAL REVIEW<sup>§</sup>

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### ABSTRACT

We elaborate, in tutorial fashion, a theoretical framework that originated from observations of phase transitions in human movement coordination. Based upon theories of self-organization and pattern formation in dissipative dynamical systems (in particular, Haken's [1983] synergetics), this theoretical but operational language is aimed at understanding the behavioral patterns produced by biological systems. The key concepts are the identification of *collective variables* (or *order parameters*) for behavioral patterns and the determination of their *dynamics* obtained through study of the *stability* (and *loss of stability*) of behavioral patterns. Methods for calculating stability measures are defined and discussed (e.g., fluctuations, relaxation times, time scale relations). Such measures, when obtained in experiment, yield results that agree with theoretical predictions. *Behavioral information* is shown to contribute to the pattern dynamics, attracting the system toward the (e.g., environmentally specified, intended, learned) behavioral pattern. Such behavioral information is

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defined in the same space as the collective variables that characterize the patterns and thus is meaningful and specific to biological functions or tasks. Although *dynamic pattern theory* (e.g., Kelso & Schöner, 1987; Schöner & Kelso, 1988a) was formulated in the context of movement coordination, other experimental systems (e.g., speech), other types of behavioral patterns (e.g., locomotory gaits, action-perception patterns), and other levels of description (e.g., neuronal activity) are accessible to this level-independent approach.

#### PROLOGUE

This chapter is dedicated to the genius of the behavioral physiologist Erich von Holst, whose unique contributions to the understanding of coordinated behavior anticipated current advances in the behavioral and brain sciences. As a result of his extensive comparative studies of locomotion (2 miles of tracings!), von Holst (1939/1973, pp. 119-120) synthesized the following rules, paraphrased for present purposes:

1. Only a certain proportion of the extremely wide range of behavioral forms is actually realized. The ones observed are distinguished from others by their greater stability.
2. This stability is expressed in the fact that with smooth or gradual alteration of internal or external conditions, periodic forms maintain themselves until a critical limiting condition is reached. Transference to another equilibrium relationship occurs—usually abruptly—which is then maintained over a particular range of conditions.
3. The stability that characterizes the periodic forms as a whole does not apply to individual temporal subdivisions, in which disequilibrium states are more likely to occur. These disequilibria are exactly balanced within the temporal unit of the entire period.
4. There is a general tendency towards transference to equilibrium states of ever-increasing stability. The degree of stability increases with the simplicity of the frequency relation-

ships. Increasing degree of complexity is accompanied by decreasing stability.

As revealed in the present chapter, von Holst's rules can now be cast in a theoretical language that has evolved over the last decade (the mathematical concepts and tools of nonlinear dynamical systems), leading to testable predictions for specific experimental model systems and deeper insights into the nature of coordinated behavior.

The human brain possesses  $10^{14}$  neurons and neuronal connections, is influenced by hundreds of active chemicals, and displays highly complex patterns of electrical activity. New concepts and tools are needed if the inherent complexity of the most complex system of all—the brain and its relation to behavior—is to be understood. Presently, there is a huge void between what a single neuron does (which we know a lot about) and what many of them do when they cooperate. Why is it crucial to discover the principles of coordination among large numbers of interacting components? The answer is that this cooperative behavior lies at the root of understanding ourselves and the world we live in—how we touch, see, hear, plan, and act. Such fundamental behavioral functions depend on temporally coherent functional units distributed throughout different regions of the brain and are not elucidated by standard methods.

When we use the word *how* in this chapter, we mean the discovery or identification of laws or principles of coordination at a chosen level of observation (e.g., kinematic, muscular, neuronal). Given that the nervous system is high-dimensional, as is the environment within which nervous systems have evolved, laws of coordination are expected to be instantiated at numerous scales of description. Further, it is possible that the long sought for link between neuronal activities (microscopic events) and behavior (macroscopic events) resides in collective effects (pattern formation) at the microscopic level that create macroscopic order (and disorder). Thus, we view the problem of coordination as continuous with efforts to understand pattern formation in complex systems with many interacting components, in particular, Haken's (1975; 1983) *Synergetics*, a theory of self-organization in nonequilibrium systems. In synergetics, methods have been found to compress system complexity, for example, in various physical, chemical, and biochemical systems that contain many degrees of freedom, to only

ie or a few degrees of freedom, the so-called order parameters, whose dynamics (equations of motion) are low-dimensional (Haken's slaving principle). The beauty of the resulting dynamics, which are in general nonlinear, is that they give rise to complex behavioral patterns, including multistability, multiple patterns, flexibility, and even deterministic chaos. Thus, the two seemingly ametrically opposed views in science of surface simplicity arising from deep complexity and surface complexity arising from deep simplicity (Yates, 1987) are both, in fact, part and parcel of nature's sign for complex systems (Kelso, 1988).

Of course, the amount of information necessary to describe the individual states of neurons and muscles is very large, and ways must be found to select the relevant quantities to compress the amount of information (see also Haken, 1987). In the case of large-scale neural systems like the brain, this is difficult to do if we treat the brain as a general purpose machine capable of producing arbitrary outputs to arbitrary inputs. An alternative strategy, exemplified here, is to treat the brain more as a "special purpose device" (Haken, 1977) that temporarily self-organizes for particular tasks (e.g., Kelso & Scholz, 1985; Schöner & Kelso, 1988a; Sejnowski, Koch, & Churchland, 1988). Many neurons, muscles, and joints must cooperate in the performance of behavioral functions. Evolving patterns of activity among these components may best be understood with respect to their functional significance for the organism. Thus, we argue here, it is *when the nervous system is involved in performing certain behavioral tasks that one sees it living* in the low-dimensional space of order parameters. This is where the laws of coordinated behavior lie.

Elsewhere, building on the concepts and tools of synergetics, we have elaborated an operational approach to biological coordination that embraces both theory and experiment (Kelso & Scholz, 1985; Kelso & Schöner, 1987, 1988; Kelso, Schöner, Scholz, & Haken, 1987; Schöner & Kelso, 1988a, 1988b, 1988c). Rather than describe this "dynamic pattern" theory again, here we adopt a more tutorial attitude intended to (a) communicate the essentials of the approach, which involves a synergy between theory, computation, and experiment; and (b) demonstrate the broader significance of the approach for understanding coordination in different experimental systems and at different levels of description. Moreover, we show that dynamical laws are fundamental, in the sense of the need for their prior identification, if certain *essentially* biological and

psychological functions such as learning, adaptation, and intentional behavioral change are to be better understood. Because the language we use may be new to students of motor coordination we present the essentials in a series of questions and answers, a kind of dialogue. This language, we stress, is not at all a question of neologisms but rather emphasizes the operational character of the approach, which requires that all theoretical constructs must explicitly relate to experiment. In this way, we think, scientists who study coordination, the collective behavior of many interacting components, at different levels of description may communicate in an unambiguous fashion. Relatedly, as we shall show, the language of nonlinear dynamics provides a way of linking levels of description and many different phenomena.

*Q. In any given experimental system, how do you identify the relevant degrees of freedom?*

A. In biology, we don't know what the relevant degrees of freedom are, a priori. In this sense, engineering or robotics approaches are not especially helpful. The engineer designs the system and thus can explicitly define the degrees of freedom in terms of the type of action possible for a given rigid segment. Depending upon the motion required and the number of orthogonal planes in which the motion is executed, the degrees of freedom for a joint may vary from one to three. It is very tempting to view the body as a collection of mechanical linkages in which one rigid part is connected to another with various restrictions on the motions possible.

As the name implies, an aim of dynamic pattern theory is to identify the degrees of freedom corresponding to patterns, where the word *pattern* is viewed always in terms of a particular function or task. These patterns are not fixed by the conventions of mechanics rather, they are flexibly assembled in order to satisfy certain boundary or task conditions. In speech, for example, there is good evidence for certain constriction points (e.g., the closing of the lips, preserving a tongue-palate relationship) that are crucial if a given sound is to be communicated (e.g., a /b/, a /p/, an /f/, or a /z/). The *relevant collective* variable then, around which the many components (e.g., jaw, lips, tongue, velum, and pharynx) are self-organized, is a task- or sound-specific constriction point (Abbs, Gracco, & Cole, 1984; Kelso, Tuller, & Fowler, 1982; Kelso, Tuller, Vukobratovic, & Fowler, 1984; see also Saltzman & Kelso, 1987).

A key to the precise definition of degrees of freedom corresponding to patterns is to find phase transitions, that is, situations in which the system's behavior changes qualitatively. As one varies a task dimension (in psychology we might say "manipulates an independent variable," although, as we shall see, that language is not appropriate for a variety of reasons), many measurable quantities may change smoothly or stay the same. Qualitative change, however, allows one to clearly distinguish one pattern from another and enables one to specify which dimension of the pattern is relevant. In addition, differential effects of the transition make it possible to study the relative stability of different patterns. (For another approach, which, however, does not study transitions as a tool for understanding coordinated movement, see Kugler & Turvey, 1987.) From the dynamic pattern view, the discovery of a phase transition enables one to identify the *order parameter*, or collective variable, corresponding to the pattern itself, and the *control parameter* or *parameters* that lead the system through these patterns. Control parameters, in the dynamic pattern approach, are *unspecific* to the resulting patterns; they carry no information whatsoever about the pattern that emerges. Under continuous changes in a control parameter, patterns may emerge spontaneously. In fact, this is a signature feature of self-organization. That is, patterns arise solely as a function of the dynamics of the system. There is no specific ordering influence from the outside and no homunculus sitting inside.

It is always crucial to establish theoretical notions in a concrete, experimental situation. Thus, the discovery of phase transitions in studies of human bimanual coordination formed the cornerstone of dynamic pattern theory and more generally of the synergetic approach to biology (Haken, 1987). The observations were as follows: Kelso (1981; 1984) had subjects rhythmically move their index fingers or hands under two initial conditions, one in which limb segments move in the same direction and electromyographic (EMG) activity of pertinent muscles fires synchronously (homologous muscles contracting *in-phase*) and an anti-phase condition in which homologous muscles contract in an alternating fashion. Through the use of a pacing metronome, frequency of oscillation was systematically increased. Figure 1.1 shows a time series when the hands were prepared initially in the anti-phase mode. Obviously, at a certain critical frequency, switching occurs spontaneously from the anti-phase to the in-phase mode. This switch is re-

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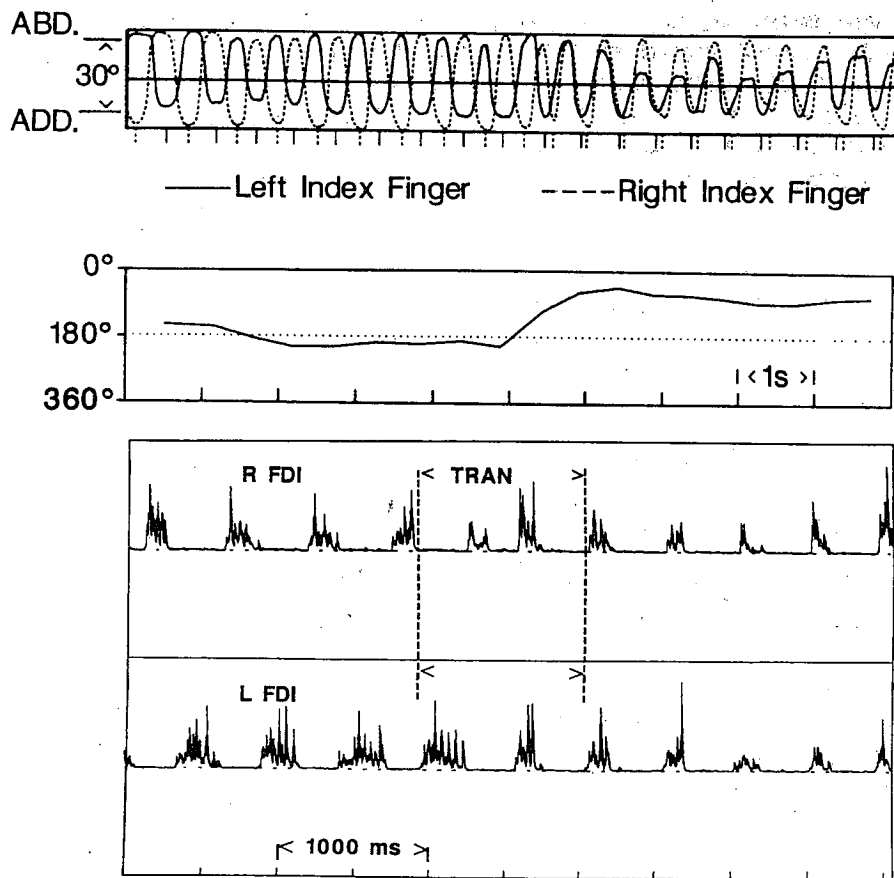


Figure 1.1. Top: time series of left and right finger position; middle: point estimate of relative phase; bottom: EMG recordings of first dorsal interossei muscle for the left and right fingers (integrated and rectified).

flected in the point estimate of relative phase in the top graph in Figure 1.1 as well as on a different level of description (EMG) in the bottom graph of Figure 1.1. No switching in the reverse direction occurs when the subject starts in the in-phase mode. Thus, although there are two stable patterns for low frequency values, only one pattern remains as frequency is scaled beyond a critical region. This transition behavior can be monitored by calculation of the relative phase between the two fingers. A point estimate of relative

phase is the latency of one finger with respect to the other finger's cycle, as determined from peak-to-peak displacement. A continuous estimate of relative phase (i.e., at the sampling rate of 200 Hz) can be obtained from the phase plane trajectories of both fingers (the velocities may be obtained by a central difference numerical differentiation procedure). When the finger oscillations are normalized to the unit circle, the phases of the individual fingers can be obtained from the arctangent ( $\dot{x}/x$ ) if  $x$  is normalized finger position and  $\dot{x}$  the velocity (see Kelso & Tuller, 1987). Relative phase is just the difference between these individual phases. Often the relative phase fluctuates before the transition and stabilizes thereafter (e.g., Kelso, 1984; Kelso & Scholz, 1985; see following discussion of fluctuations).

What is the relevant degree of freedom in this case—or in the language of dynamic patterns, what is the order parameter or collective variable? From our discussion of phase transitions, the relative phase,  $\phi$ , is a suitable candidate because (a)  $\phi$  characterizes all observed coordinative patterns; (b)  $\phi$  changes abruptly at the transition and is only weakly dependent upon parameters outside the transition; and (c)  $\phi$  has very simple dynamics in which the ordered, phase-locked patterns correspond to attractors (we will define the term *attractor* shortly). Because the prescribed frequency of oscillation, manipulated during the experiment, is followed closely (i.e., is not affected by the relative phase) and because frequency drives the system through the collective states, frequency may be considered the control parameter.

In summary, we want to stress again that the phase transition methodology allows one to identify relevant degrees of freedom. Phase transitions represent singular boundaries that separate, as it were, different realms of existence, in our case, movement patterns. Nonequilibrium phase transitions are a universal feature of all complex systems; oscillating fingers are simply a window into establishing this fact for biological coordination.

*Q. Once the relevant degrees of freedom or order parameters are found empirically, how do you model?*

*A.* A first step is to provide a mathematically accurate description of the main qualitative features of one's data. The key idea is to map observed patterns onto attractors of a dynamical model. The meaning of *dynamical* here has nothing to do with forces or masses in the



conventions of mechanics; rather, it refers to the temporal evolution of (in our case) a collective variable, that is, how this variable changes or stays the same as time flows (formally, the flow of a vector field). Given that we have identified a relevant degree of freedom,  $x$ , characteristic of the pattern, underlying the dynamic pattern view is the assumption that  $x = x(t)$ , where  $t$  is time, obeys a dynamical law:

$$\dot{x} = f(x_t, \text{parameters}, \text{noise}) \quad (1)$$

For a large class of functions  $f$ , special solutions of Equation (1) exist called *attractors*. By definition, an attractor is asymptotically stable; that is, all neighboring solutions of Equation (1) converge in time to the attractor solution. Nonequilibrium systems generally obey dissipative dynamics, the word *dissipative* meaning that many independent trajectories of the system with different initial conditions eventually converge on a certain limit set, the attractor. The simplest attractor type is a stable fixed point, that is, a constant solution of Equation (1) to which all neighboring trajectories converge. Another important attractor type in biology is the *limit cycle*, a stable periodic solution of Equation (1). Many more complicated attractor types exist (see also Newell et al., this volume), and their identification has proved significant in many branches of science (see Campbell, 1987; Kelso, Mandell, & Shlesinger, 1988) and medicine (see Koslow, Mandell, & Shlesinger, 1987). Attractors play a key role in the modeling process because the behavior of the collective variable in time (the dynamics of the collective variable) may be mapped onto attractors, the layout of which may be altered as a control parameter is changed continuously.

For the hand experiments of Kelso and colleagues, Haken, Kelso, and Bunz (1985) were able to determine the dynamics of relative phase,  $\phi$ , from a few basic postulates. First, the observed stationary states of  $\phi$  at  $0^\circ$  and  $\pm 180^\circ$  are modeled as point attractors. This is a minimality strategy in which only the observed attractor type appears in the model. *It cannot be overemphasized that these are point attractors in the space of collective variables*, that is, of the system's relevant degrees of freedom. Second, the model must reproduce the observed bifurcation; that is, two patterns are available below a critical frequency, a condition called *bistability*, whereas only one is stable above the critical point (*monostability*). Third, due to the angular character of  $\phi$ , the dynamics have to be  $2\pi$  periodic. Another way of saying this is that because  $\phi$  occurs only under cosine

or sine functions, the properties of the physical system must not change when  $\phi$  is replaced by  $\phi + 2\pi$ . Fourth, both hands are assumed to play a symmetric role; that is, the behavior of the system does not depend on the way we label the right hand and the left hand. This means that the model is symmetric under the transformation  $\phi$  to  $-\phi$ . This assumption fits the data well; evidence for hand preferences in the bimanual experiments is weak at best.

In synergetics, the equations for order parameters are often of the form

$$\dot{\phi} = - \frac{\partial V}{\partial \phi} \quad (2)$$

where  $V$  is the so-called potential function. Following this strategy, the simplest and most general model obeying the four postulates is a potential,  $V$ , that is a superposition of two cosine functions:

$$V = -a \cos(\phi) - b \cos(2\phi) \quad (3)$$

which is an explicit model of the dynamics of relative phase with two parameters,  $a$  and  $b$ .

The behavior of the system described by Equations (2) and (3) can be readily visualized if  $\phi$  is identified with the coordinate of a particle that moves in an overdamped fashion in the potential,  $V$ . When the total superposition (Equation 3) is taken and the ratio  $b/a$  is changed, a whole series of potential fields can be traversed (see Schöner, Haken, & Kelso, 1986, for how the parameters  $a$  and  $b$  are calculated from real data). Now prepare the system, as in the state shown by the black ball in Figure 1.2 ( $\phi = \pm\pi$ ). Decrease the ratio  $b/a$ , which corresponds to increasing the experimental frequency. At a critical value of the parameters, the black ball falls to the lower minimum at  $\phi = 0$ . This corresponds to the transition from the anti-phase (antisymmetric:  $\phi = \pm\pi$ ) state to the in-phase (symmetric:  $\phi = 0$ ) state. When frequency is further increased ( $b/a$  tends to 0), the hand movements remain in the symmetric pattern. Note also that if the system is prepared in the symmetric pattern and  $b/a$  is decreased, no transition to  $\phi = \pm\pi$  occurs. Also, following a transition to  $\phi = 0$ , if  $b/a$  is increased again (corresponding to a decrease in experimental frequency), the system remains in that state. This hysteresis phenomenon is well known in many physical and biological systems and was also a feature of the hand experiments.

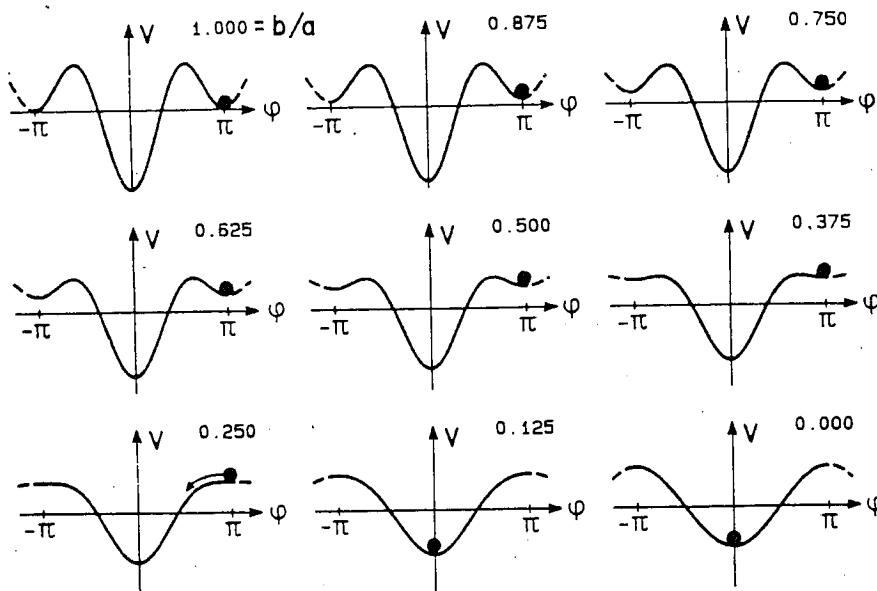


Figure 1.2. The potential (3) as the ratio  $b/a$  is changed (numbers refer to  $b/a$ ). The system is initially prepared anti-phase ( $b/a = 1.0$ ). As  $b/a$  decreases, the little ball illustrates the system's transition to in-phase ( $b/a = 0.0$ ) cycling behavior. Note: from "A Theoretical Model of Phase Transitions in Human Hand Movements" by H. Haken, J. A. S. Kelso, and H. Bunz, 1985, *Biological Cybernetics*, 39, p. 150. Copyright 1985 by Springer-Verlag. Reprinted by permission.

Q. Your theoretical strategy enables you to provide a compact description of the hand experiment results, but how do you really know that stability and change of biological movement patterns correspond to nonequilibrium phase transitions?

A. The answer is that we don't know in advance and that we have to find out. Certainly, not all changes correspond to phase transitions. For example, as frequency was increased in the bimanual experiments, there was often a parallel, smooth decrease in movement amplitude in both fingers (see Kay, Kelso, Saltzman, & Schöner, 1987). However, such changes are best viewed as parametric. On the other hand (pardon the pun), in systems close to transition points, certain specific phenomena are predicted to occur jointly. One of

These predictions involve fluctuations around the mean state of the collective variable. These fluctuations arise from the dynamics of numerous subsystems to which the system is coupled. The collective effect of these underlying processes acts as a perturbation in the form of noise. As the system approaches a critical point, one may observe an accompanying increase in these fluctuations—so-called *enhancement of fluctuations*—reflecting the growing inability of the system to maintain a particular pattern. In the critical region itself, the system briefly displays transient behavior, in which no definitive pattern is apparent. The system then evolves to a new or different pattern, apparent from the new value of the collective variable. The switch to a new pattern is accompanied by a marked decrease in fluctuations, signifying that the transition to a new stationary state is complete (cf. Prologue).

At values of the control parameter where fluctuations are minimal, the pattern is considered to be more stable than in control parameter regimes where higher fluctuations are observed. Stability, therefore, is not just an intuitive descriptive label; rather, it is a well-defined concept that is central to dynamic pattern theory (for further discussion see Kelso et al., 1987; Schöner & Kelso, 1988a). Stability serves a dual purpose in linking theory and experiment: it not only does it characterize the states in which the system resides, it also *loss of stability* in the order parameter is hypothesized to be the chief mechanism that effects a change of pattern. More explicitly, fluctuations may be considered stochastic forces, acting as continuously applied perturbations that drive the system away from its present state. In the bimanual experiments, these fluctuations were measured as the standard deviation of the collective variable, relative phase. In other nonequilibrium systems, other observables such as the output of laser light or the molecular concentration in chemical reactions undergo large fluctuations (see Aiken, 1983, for many examples).

Kelso and Scholz (1985; see also Kelso, Scholz, & Schöner, 1986) analyzed the mean relative phase and its standard deviation in each of the two coordinative patterns as frequency (a control parameter) was increased. When the motions were prepared initially in the symmetric pattern, the mean relative phase and standard deviation remained relatively constant. However, clear enhancement of fluctuations both before and during the transition were observed when the movements were prepared anti-phase. Furthermore, after the switch from the anti-phase to the in-phase pattern, fluctuations de-

creased dramatically to levels comparable to the symmetric, in-phase condition. In Figure 1.2, it is quite easy to see what enhancement of fluctuations means. Any small fluctuation, when the attractors are well defined (top left), will be quickly damped; that is, the effect of fluctuations in this parameter regime is small. However, the same fluctuations, when parameters flatten the minimum at  $\phi = \pi$ , will be seen to be greatly amplified.

A second prediction of dynamic pattern theory and synergetics in general that further characterizes the differential stability of the attractors at different values of a control parameter concerns *critical slowing down*. Simply stated, when a system is close to a transition point, the system reacts more slowly to external perturbations than it does when it is far removed from the critical point. If a small perturbation is applied to the system, driving it away from its stationary state, the time it takes for the system to return to that state, the local relaxation time ( $\tau_{rel}$ ), is a measure of the stability of the attractor. The smaller  $\tau_{rel}$ , the more stable the attractor. Obviously, as the phase transition regime is approached, enhanced fluctuations should be reflected in a parallel increase in the duration of  $\tau_{rel}$  (actually, they grow as a square root function of the relaxation times; cf. Scholz, Kelso, & Schöner, 1987; Schöner et al., 1986). As well, once the critical point is crossed, the sharp decrease in fluctuations that follows the emergence of a new pattern should be accompanied by a decrease in local relaxation time. In terms of the potential of Figure 1.2, critical slowing down is reflected in a flattening of the pretransition minimum (as  $b/a$  is decreased and the transition regime is approached). Thus, when the little ball is pushed away from the minimum by a perturbation, it takes longer to return to the minimum in this flattened state than it does when the shape of the potential is steeper (i.e., at lower values of the control parameter where restoring forces are large, or after the transition is complete and the system resides in a new, well-articulated basin of attraction. The attractor basin is defined as the set of all initial points from which trajectories converge to a given attractor.) At even higher values of the control parameter, the initial minimum disappears altogether, and the little ball falls to the sole remaining basin of attraction.

The critical slowing down prediction of synergetics was tested in the bimanual paradigm (Scholz & Kelso, in press-b; Scholz et al., 1987). A torque pulse (50 ms duration) perturbed one of the index fingers as they moved at different frequencies. This pulse acted as a perturba-

tion that moved the bimanual pattern temporarily away from its prepared state and enabled calculation of the time to return to the initially prepared pattern. The results were extremely consistent with predictions: (a) Except for the lowest frequencies, the relaxation time in the anti-phase mode was consistently higher than in the in-phase mode; and (b) as the system approached the transition, the relaxation time increased in the anti-phase mode but remained constant or decreased in the in-phase mode. Thus, through the use of an entirely different experimental observable, support for a nonequilibrium phase transition in biological coordination was provided. In sum, observations of critical slowing down and enhancement of fluctuations are typical features of *self-organization* in synergetic systems. New states evolve without specific influence from the outside; that is, the control parameter does not anticipate or indicate the new state but rather creates the necessary conditions for the system to acquire it.

When collective variables for patterns are identified, relaxation times and fluctuation measures are well defined and open to observation. As we have noted, the key step is to link such measures to the concept of stability. Time scales play a crucial role in this regard, that is, in the interpretation of observed patterns as attractor states of a dynamical system. Up to now, we have introduced local relaxation time,  $\tau_{rel}$ , as a measure of the time it takes the system to relax to an attractor once it is nearby. But two other time scales in addition to  $\tau_{rel}$  are important: (a) observation time ( $\tau_{obs}$ ), the typical time scale on which the experimenter observes the system in a given preparation and over which statistical averages are performed; and (b) equilibration time ( $\tau_{equ}$ ), the time it takes the system to reach a stationary probability distribution, or stationary state, from a typical initial distribution. In the bimanual case,  $\tau_{equ}$  is the time it takes the system to travel from one basin of attraction, for example, at  $\phi = \pm\pi$ , to the other basin of attraction at  $\phi = 0$ . In order to interpret observed states as (local) attractor states, the following time scales relation must be fulfilled:

$$\tau_{rel} \ll \tau_{obs} \ll \tau_{equ} \quad (4)$$

Here, the time it takes the system to relax back to its stationary state is shorter than the time over which the system is observed (e.g., for a given value of the control parameter) and far less than the time necessary to reach its most stable state. In terms of Figure 1.2, this relation means that the rolling ball may be pushed away from

its present minimum, but the push is not enough to force it over the potential hill. That is, the system relaxes to its (local) stationary state on the observed time scale. However, as a state loses stability, its local relaxation time increases until, at the phase transition, the time scales relation (4) is violated, and switching occurs. The nature of the transition (e.g., whether critical fluctuations will be observed) depends on another time scale, the time scale of parameter change,  $\tau_{\text{par}}$ . This time scale reflects the fact that in biological systems, the control parameter that brings about the instability is itself changing in time. For example, if

$$\tau_{\text{rel}} \ll \tau_{\text{par}} \ll \tau_{\text{equ}} \quad (5)$$

then the system remains at a control parameter value much longer than the time taken to return to its present state after being perturbed, allowing the system to relax to a locally stable state. In such cases, typical features of critical phenomena (enhancement of fluctuations and critical slowing down) are predicted. Thus, in our experimental system, if time scales relation (5) holds, it is possible to maintain a particular pattern even as fluctuations increase, and a transition is observed only as the old state becomes unstable. If, however,

$$\tau_{\text{rel}} \ll \tau_{\text{equ}} \ll \tau_{\text{par}} \quad (6)$$

meaning that the control parameter is fixed at a value that is quite long relative to the time it takes the system to find a global state of stability, we may see no enhancement of fluctuations because the system seeks out the lowest potential minimum before the old state actually becomes unstable. One can readily see the general implications of these time scale relations for the design of experiments. In the bimanual work, because  $\tau_{\text{par}}$  and  $\tau_{\text{obs}}$  are of the same order in the experiment, we expect time scales relation (5) to hold up to the transition. This requires us to differentiate two parameter regimes: the noncritical regime in which the system is stationary in the sense of relation (5), and the critical regime in which the system exhibits transient behavior. In these regimes, the full stochastic dynamics of Equation (3) can be solved numerically (Schöner et al., 1986) with pretransitional information about the standard deviation of relative phase and relaxation time of the anti-phase mode. The stochastic model accounts very well for the transient behavior without adjusting any parameters. Moreover, we should emphasize

that all time scales are measurable. In the case of  $\tau_{\text{obs}}$  and  $\tau_{\text{par}}$ , measurement is often quite obvious; for  $\tau_{\text{equ}}$ , several measurement techniques exist. One direct measure is called *mean first passage time*, which is the average length of time before the system first changes state. In the bimanual experiments, mean first passage time was determined directly using experimental data (Scholz et al., 1987), enabling a direct test of relation (5) and its breakdown at the phase transition. Switching indeed occurred as the time scale relation was violated. Thus, time scale relations that govern the switching dynamics among collective states are directly observable and become even more important when other essentially biological features such as learning (Schöner & Kelso, 1988f) and development (Thelen, Kelso, & Fogel, 1987) are considered.

The stochastic version of Equation (3) (Schöner et al., 1986) contains another novel feature that pins down the nonequilibrium phase transition interpretation of stability and change in patterns of coordination. This feature is the duration of the transient from the anti-phase state to the in-phase state—which we call *switching time*. Due to the stochastic aspect of the dynamic model, switching does not occur as soon as the critical frequency is reached. Instead, during the transition, the probability density of relative phase, initially concentrated at  $\phi = \pm 180^\circ$ , flows to  $\phi \approx 0^\circ$  and accumulates there until the "new" peak at  $\phi \approx 0^\circ$  is dominant and stationary. The model predicts the duration of this process in terms of both its mean and distribution. Consistent with our operational approach, switching times were also extracted from experimental data (Scholz & Kelso, in press-b; Scholz et al., 1987). In most cases, they were easy to calculate as the time between the relative phase value immediately before the transition and the value assumed immediately following the transition. The striking agreement between theoretical prediction (Schöner et al., 1986) and empirical data is particularly interesting because it shows that the switching process itself is quite closely captured by the stochastic dynamics of Equation (3) with noise added. The language of phase transitions is thus adequate for understanding the present phenomenon and opens the way to explore others (see Kelso & Schöner, 1987, for further examples).

Q. Are there other ways to identify attractors of the collective variable dynamics?



A. A technique that has proven useful in our experimental data is a sampling technique known as a return map. Here we use the information provided by relative phase values (point estimate) to determine whether a relation exists between one relative phase value ( $\phi_n$ ) and the next ( $\phi_{n+1}$ ). Essentially, this method tests for a deterministic structure in the dynamics by grouping discrete relative phase values into pairs ( $\phi_n, \phi_{n+1}$ ) and plotting each as a point over time. If the relative phase values are  $2\pi$ -periodic, then each pair plots the same point, a pattern suggesting the presence of a point attractor in this particular state space. Similarly, a limit cycle would appear as the same points revisited in a particular order. At the opposite extreme, a purely random process would appear as a scatter of points reflecting the lack of any history in the succession of values (see, e.g., Shaw, 1984).

Before giving an example, however, we emphasize that discreteness may be imposed in a number of ways. If we consider a 3-D trajectory in the space of  $(x, \dot{x}, t)$ , we may sample, or "strobe," the trajectory at a specified time within each cycle of its path. Essentially, this sampling defines a plane perpendicular to the  $(x, \dot{x})$  plane. Each time the trajectory passes through this plane, a point is plotted ( $\phi_n, \phi_{n+1}$ ), the successive values of which enable us to construct a return map. In our example, which uses peak-to-peak relative phase, time itself is not the sampling variable. By choosing the peak displacement of each cycle, we define a sampling surface perpendicular to the  $(\dot{x}, t)$  space whose shape is a deformed plane. In other words, the point at which we sample will vary with changes in peak displacement values. Otherwise, if the peak displacement value is exactly the same in each cycle, then the sampling plane is uniform. Notice also that if the oscillatory components are perfect sinusoids, then peak values define uniform perpendicular planes in both the  $(x, \dot{x})$  and  $(\dot{x}, t)$  space because peaks define constant time and displacement values.

Shaw (1981) draws attention to three factors on which the success of this method depends. First, the important attractor properties are topological; that is, almost any set of coordinates that can be manufactured will suffice to describe the attractor. Second, the system is nearly deterministic for short times; that is, the point-to-point structure of the attractor is well defined. Third, the dimensionality of the attractor is in fact small enough to be tractable (see Kay, Saltzman, & Kelso, 1988, for application of dimensionality calculations to motor behavior; also Kay, in press).

Figure 1.3 is a series of relative phase return maps for the cycling behavior of the right arm and right leg of a human seated in an experimental apparatus that allows all four human limbs to cycle up and down in the sagittal plane (Jeka & Kelso, 1988; Kelso & Jeka, in preparation). By convention, relative phase values close to  $180^\circ$  correspond to the arm moving up as the leg moves down; relative phase values around  $0^\circ$  correspond to both limbs moving up and down in the same direction. Cycling frequency was scaled as in previous bimanual experiments. Relative phase values for cycle  $n$  are plotted on the horizontal axis versus relative phase for cycle  $n+1$  on the vertical axis. Each of the four maps represents two of eight successive frequency plateaus (from 1.25 to 3.00 Hz in steps of 0.25 Hz). The first two frequency plateaus appear in the upper left map, with following plateaus graphed in a clockwise direction. In each map, the phase values of the first plateau pair are plotted with the white squares, and the second plateau appears as black triangles.

The convention we have adopted to plot phase values in these maps is that points appearing in any of the four corners of this coordinate plane denote approximately equivalent states that vary around  $0^\circ$  (or  $360^\circ$ ). They differ only in that when the arm reaches its peak displacement slightly ahead of the leg, the relative phase values are slightly less than  $360^\circ$ , whereas if the arm is slightly behind the leg, relative phase values are slightly greater than  $0^\circ$ . Points lying on either end of an imaginary  $45^\circ$  line from the origin are the special case in which  $\phi_n = \phi_{n+1}$ , that is, when the lead-lag relationship remains the same in successive cycles. Points at either end of the opposing diagonal occur when the lead-lag relationship between limbs changes in successive cycles. All points clustered around an anti-phase ( $180^\circ$ ) value, however, will appear grouped around the center of the coordinate plane.

Beginning with the upper left map, one sees a cluster of points grouped around a  $180^\circ$  anti-phase relationship, corresponding to the initially prepared pattern. The arrows reflect the fact that signs of instability are already emerging by the second frequency plateau, as relative phase briefly wanders toward an in-phase pattern, before returning to an anti-phase relationship, by the end of the plateau. The next map shows even more transient behavior, as points travel in a clockwise direction toward the origin and then around three corner points corresponding to an in-phase pattern

 $\phi_{n+1}$  $\phi_n$ 

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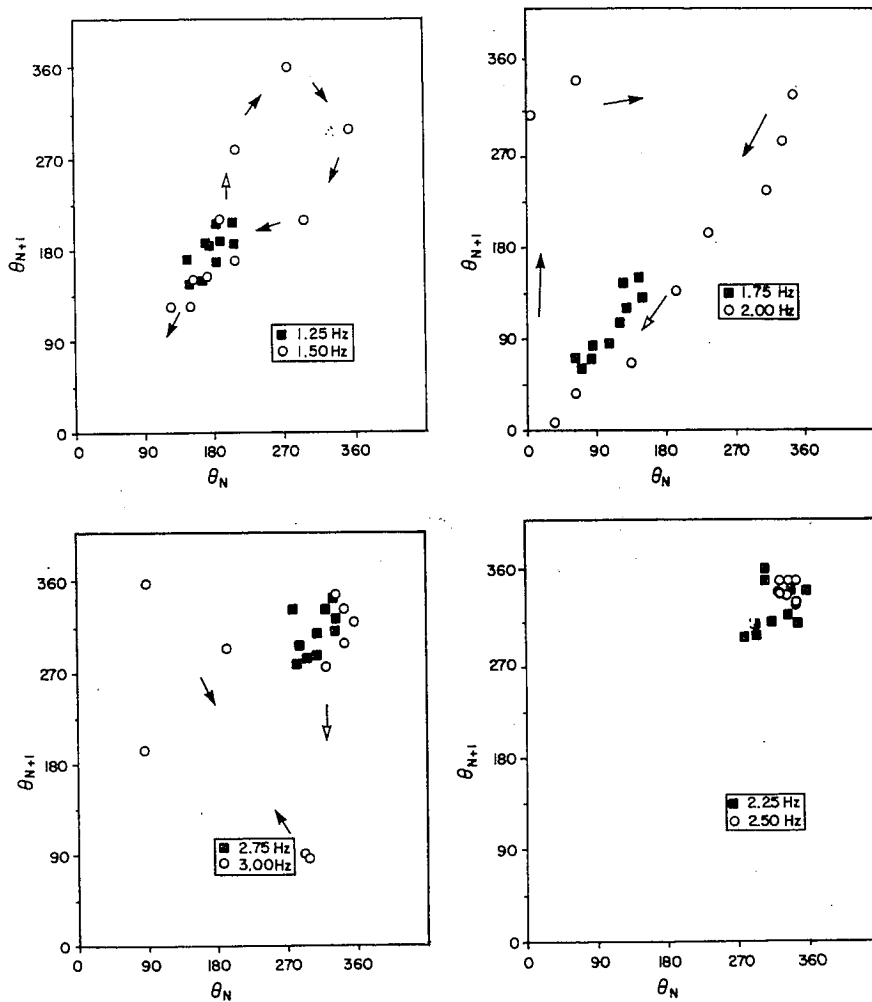


Figure 1.3. Return maps: the relative phase of cycle  $n$  (abscissa) versus cycle  $n+1$  (ordinate). Each map contains two successive frequency plateaus beginning clockwise from top left. Arrows depict the change between successive relative phase values. White arrows show the initial change from a temporary stationary value of relative phase.

with changing lead-lag relationships between the limbs. The map in the bottom right corner indicates that the transition is now complete, as points remain clustered close to 360° (in-phase) for two

complete plateaus as well as the first plateau of the last map (bottom left). The final map also demonstrates the transition created by the limbs operating at different frequencies, a transition that occurs when the leg can no longer follow the required frequency as set by a metronome. One now observes relative phase values jumping from one side of the 45° line to the other, possibly suggesting a bifurcation to limit cycle behavior in which phase values visit a number of repetitive values (states) in a specific order. We must emphasize, however, that this pattern does not prove the existence of an attractor but merely suggests that some underlying deterministic process may be driving the interaction between the limbs. Such a suggestion is legitimate only if it is supported with many more samples and a quantitative analysis (Kelso & Jeka, in preparation).

The important point illustrated by the return map, which has been discussed previously, is that a system with numerous components may be represented by a small number of parameters. We are essentially mapping a higher dimensional space (minimally, the position and velocity of each limb with possible coupling degrees of freedom) onto a low-dimensional map from which one can observe transitions in the collective variable, relative phase, and the emergence of new, stable phasing relationships. Such maps allow one to explore generalities that may apply to many systems that would otherwise be difficult to compare. For example, the well-known logistic map (e.g., May, 1976),  $f(x_{n+1}) = \lambda x_n (1 - x_n)$ , possesses a single parameter,  $\lambda$ , which has certain universal properties—known as Feigenbaum constants (Feigenbaum, 1983)—that have been quantitatively measured in a number of experimental systems. Such one-dimensional maps are capable of modeling the complexity of behavior typically seen in high-dimensional systems and illustrate again how important it is to identify the relevant degrees of freedom in complex systems.

We must stress that graphical characterizations indicative of biological attractors are not without historical precedent, even though the significance of such characterizations may not have been realized when they were first introduced. For example, Erich von Holst (1939/1973), whose work on the oscillatory rhythms of fish fins was one of the earliest efforts to characterize stability in a biological system, formulated an empirical technique known as the time and speed tables. Researchers have used this technique more recently to characterize cardiac rhythms in cats (Reid, 1969) and cellular bursting rhythms in the nervous systems of lobsters (Ayers &

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Silverston, 1979; see also Stein, 1976). Two rhythms from von Holst's own work are shown in the top half of Figure 1.4. Even though both signals show periodicity, there are also nonuniform features to the lower signal. The question is whether these changes connote an influence between the two rhythms or whether they are merely "noisy" fluctuations in otherwise unrelated physiological signals.

To distinguish such possibilities, von Holst (1939/1973) first recorded the "relative phase" of the two rhythms on a cycle-to-cycle basis. He then measured the half-cycle duration for both the upstroke (fin movements to the right) and the downstroke (fin movements to the left) of each cycle. The result of these measurements appears in the bottom half of Figure 1.4, which shows the time table for the rhythms directly above it, now with cycle time normalized around a mean value. Relative phase is plotted on the abscissa versus the individual cycle time on the ordinate. The *x* curve corresponds to upstroke cycle times, and the *o* curve represents downstroke cycle times. We show only the time table here because the speed table is conceptually identical but uses average velocity within a half cycle instead of the half-cycle duration.

The time table reveals that the duration of a particular half cycle is dependent upon the momentary phase relationship between the two rhythms. This interaction is revealed by the behavior of points surrounding the intersection of the horizontal mean half-cycle duration line and the *x* curve, where the *x* curve has a maximum negative slope. Any point one chooses just below the line on the *x* curve represents a half-cycle duration that is slightly shorter than its mean value. As a consequence, the *next* half-cycle is brought closer to a relative phase relationship with the uniform rhythm, resulting in a duration that is now greater than the mean value. Again, the next upstroke is shifted further away from the vertical midline, this shift resulting in a smaller half-cycle duration, and so on. Thus, the intersection of the *x* curve and the mean value line is an "attractive" point in the sense that all combinations of cycle duration and relative phase tend toward this point (in the present language, it is a stable fixed point). The strength of that influence depends on how close the *x* point lies to the attractive point. A similar look at the point at which the *x* curve's slope is *increasing* through the mean cycle duration line reveals that it repels all nearby values (i.e., it is an unstable fixed point). It is noteworthy that the exact values of these intersection points correspond to entrained states in which

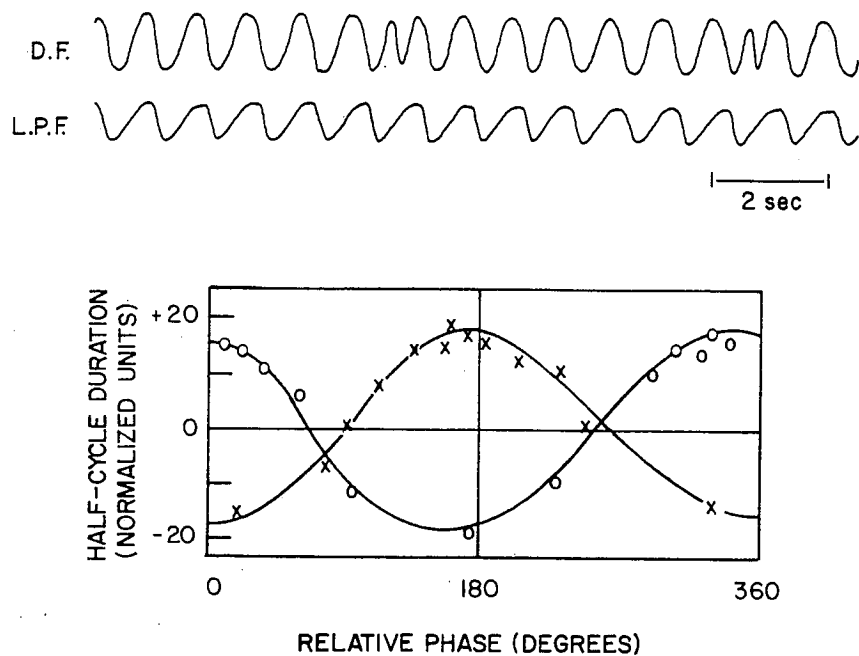


Figure 1.4. Time table for the rhythms recorded from the left pectoral and dorsal fin movements of *Labrus*. For each cycle, the relative phase of the two rhythms appears on the abscissa versus the percentage deviation from the mean value (at 0) of the half-cycle duration on the ordinate. The x curve depicts fin movements to the right; the o curve, fin movements to the left. *Note:* from "Relative Coordination as a Phenomenon and as a Method of analysis of Central Nervous Function" by E. von Holst, 1973, in R. Martin (Ed. and Trans.), *The Collected Papers of Erich von Holst: Vol. 1. The Behavioral Physiology of Animals and Man* (p. 59), Coral Gables, FL: University of Miami Press. Copyright 1973 by the University of Miami Press. Adapted by permission.

cycle duration and phase are locked. It is only when one moves away from these points that the nature of their influence emerges. The same analysis holds for the o curve, corresponding to downstroke cycles.

Although von Holst (1939/1973) was most interested in characterizing the coordination of irregular rhythms compared to

the extreme cases of entrained and random patterns, he actually provided a technique that characterizes stability in a qualitative yet precise fashion. In essence, he graphically identified fixed-point attractors; one that is stable and attracts all neighboring states and one that is unstable, on which the system remains only if it sits precisely upon this point (note again in the space of an adequate collective, temporal variable). Any slight deviation from the unstable fixed point leads to further divergence. To better visualize the existence of two such fixed points within the same system, consider the case of a freely swinging pendulum (Abraham & Shaw, 1982). The stable fixed point corresponds to the bottom of its circular arc (i.e., 6 o'clock) because no matter where the pendulum begins its swing, the combination of frictional and gravitational forces ensures its final position at this fixed point. Theoretically, however, there is one exception to this case, in which these same forces are perfectly balanced so that the pendulum remains perched at the very top of its swing (i.e., 12 o'clock). Intuitively, it is easy to see why this point is unstable: Even the slightest puff of wind will perturb the pendulum away from this fixed point toward its more stable partner. It is interesting to note that a simple, damped pendulum forced by a cosine function exhibits chaotic behavior; that is, a small uncertainty in the initial state rapidly makes it impossible to forecast future states (for a review, see Grebogi, Ott, & Yorke, 1987). Thus, even in seemingly simple systems, crisscrossing the border between stable and unstable regions of state space may result in enormous behavioral complexity.

*Q. In this picture, once you have identified the patterns and their dynamical laws, how do you derive these laws? I'm asking how different levels of description may be related.*

*A.* A key feature of the discussion thus far has been to characterize coordinated patterns in terms of the dynamics of macroscopic collective variables. But the strategy allows one to address the subsystems themselves and how these are coupled to produce coordinated patterns. This feature distinguishes the present operational approach from others in the human motor control literature. Yet it is entirely consistent with typical scientific procedure, namely, find the macroscopic laws first and then derive them from a more microscopic level of description. In this regard, we cannot help noting that the words *macro* and *micro* are always used in a relative sense, as is the case in science generally. For instance, the relevant micro

levels of the particle physicist, the condensed matter physicist, the chemist, the biologist and so forth are all very different.

In our experimental model system, the individual oscillatory components are the next level down from the patterns they produce when they are coordinated. One may define the dynamics of individual components using each limb's position ( $x$ ) and velocity ( $\dot{x}$ ) as collective variables, but collective now in terms of the *next* lower level of description (e.g., agonist-antagonist muscle combinations). The stable and reproducible oscillatory behavior of each hand is modeled as an attractor in the phase plane ( $x, \dot{x}$ ), in this case, a limit cycle. This step again is based on an accurate description of the experimental data. In a study related to the bimanual finger work (Kay et al., 1987), the cycling movements of individual hands were observed. Subjects rotated their hands about the wrist at six metronome-paced frequencies from 1 to 6 Hz in 1-Hz steps. Although subjects were given no explicit instructions about movement amplitude, the results demonstrated that amplitude remained roughly the same within a given frequency but decreased as frequency increased. It was found that with certain minimal assumptions, a nonlinear oscillator model captured these experimental features successfully. In mapping the observed oscillatory state onto a limit cycle, the notion of stability is once again a key feature of the theory. This can be checked by measuring the relaxation time of the individual components in a way similar to that described earlier. Along with the observed kinematic relations, relaxation time measures allowed us to determine all the individual oscillator parameters. A further assumption, that the oscillation is autonomous and not explicitly time-dependent, was tested in the perturbation paradigm using phase-resetting techniques (see Kay et al., 1988). The dimensionality of the attractor (Grassberger & Procaccia, 1983) was also calculated and corresponded to a limit cycle attractor with noise ( $D \sim 1.2$ ; see Kay et al., 1988).

How can the components with their dynamics give rise to the phase-locked coordinative modes? Obviously, their dynamics have to be coupled. Haken et al. (1985) have determined coupling structures that can account for the observed phase-lockings. The simplest model that achieves this is a van-der-Pol-like coupling of the form:

$$\ddot{x}_1 + f(x_1, \dot{x}_1) = (\dot{x}_1 - \dot{x}_2) [A + B(x_1 - x_2)^2] \quad (7)$$



$$\ddot{x}_2 + f(x_2, \dot{x}_2) = (\dot{x}_2 - \dot{x}_1) [A + B(x_2 - x_1)^2] \quad (8)$$

where  $f$  is the aforementioned oscillator function and  $A$  and  $B$  are coupling constants. The experimental observation that the kinematic relations (e.g., amplitude-frequency relation) are not significantly different between the coordinative modes and the single hand movements shows that the coupling constants  $A$ ,  $B$  are small compared to the corresponding coefficients of the oscillator function (see Haken et al., 1985; Kay et al., 1987). In spite of this, the coupling structure described by Equations (7) and (8) gives rise to the two phase-locked states. Indeed, Haken et al. (1985) were able to derive the equation for relative phase (3) from Equations (7) and (8) using the slowly varying amplitude and rotating wave approximations. These results not only provide further support for the dynamical model on the collective variable level but also rigorously establish the relation of the two levels of description. It is important to note that the coupling functions are quite unspecific to the emergent patterns of coordination because several functional forms may result in the same pattern of phase-locking (Haken et al., 1985). Furthermore, coordinative changes may also emerge in different ways. For example, Kelso and Scholz (1985) showed computer simulations in which the phase transition was effected by various combinations of coupling strength and noise level. Also, simply keeping the coupling function constant and changing only the eigenfrequencies of the component oscillators bring about changes in coordinative pattern. Thus, the collective properties of the system are attributable more to the coordinated system as a whole than to the actual coupling terms. This fact denotes a consistency across the physiological and mathematical domains; in both, different mechanisms may give rise to the same pattern (Kelso & Scholz, 1985; Schöner & Kelso, 1988a).

*Q. It might be helpful now if you would summarize the main features of dynamic pattern theory.*

A. This theory (Kelso & Schöner, 1987, 1988; Schöner & Kelso, 1988a, 1988b, 1988c, 1988d) builds upon the concepts of synergetics (Haken, 1975, 1983), a theory of self-organization and pattern formation in nonequilibrium systems. Synergetics provides a theoretical but operational language. The main idea is to view patterns of coordination, or more generally, behavioral patterns, at one's cho-

sen level of observation in terms of their nonlinear dynamics. We can summarize the theory as follows:

1. Patterns of coordination at a given level of description are characterized by low dimensional collective variables or order parameters whose dynamics are function-specific.
2. Observable (i.e., reproducible, stationary over a certain time scale) patterns of coordination are mapped onto attractors of the order parameter dynamics.
3. Biological boundary conditions (e.g., environmental, task, and energetic constraints) act as parameters on the collective dynamics in the sense that they can modify the behavioral patterns but are themselves not dependent on these patterns. A parameter that moves the system through different collective states is a control parameter in the sense of synergetics. These parameters may be quite unspecific to the resulting behavioral patterns.
4. Fluctuations determine not only the stability of the observed pattern but also different time scales (global and local relaxation times). Time scale relations govern the switching dynamics among multiple coordinative patterns and account for observed multistability, that is, coexistence of several patterns under the same conditions.
5. Loss of stability leads to switching of pattern, and switching is governed by stochastic order parameter dynamics.
6. If coordinative patterns are thus characterized at different levels of description, these levels may be related without introducing additional concepts.

Points 1 to 3 provide a framework through which a dynamic pattern description may be obtained. These steps must be taken for any particular experimental system at any level of description in order to give the concepts a concrete meaning. Once a consistent description in the sense of Points 1 to 3 is reached, Points 4 and 5 enable one to test crucial predictions of the theory. It is important to emphasize that the linkage among levels of observation (Point 6) is possible only if a dynamic pattern analysis is available on both levels in question.

Q. Now that you've provided the building blocks, as it were, of your approach, what further extensions are possible?

A. Although dynamic pattern theory was originally formulated in the context of patterns of movement coordination, other behavior patterns, functions, and experimental systems (a few examples follow) are open to analysis. There are really two cornerstones to the whole approach that make these extensions possible. The first is the necessity of identifying, in any given problem, the order parameter dynamics in the absence of any specific parametric influence. We refer to these dynamics as *intrinsic dynamics*, which simply means that the patterns arise as a result of *nonspecific* changes in control parameters (see Point 3). Nothing about "hard wiring" of these intrinsic dynamics is implied. However, the main point is that one has to discover the intrinsic dynamics to know *what* is modifiable, by the environment, by intentions, or by learning, for example. Once one knows the patterns and their dynamics, one can begin to talk rigorously of what it is that is changed (or indeed changeable) by *specific* parametric influences.

The specific parametric effects due to environmental requirements, intentional or purposeful needs, tasks to be learned, and so forth allow us to introduce the second, not yet discussed, cornerstone of the theory, namely, the concept of *behavioral information*. Such information may be expressed in the form of required behavior patterns, that is, as part of the dynamics that attracts a behavior pattern (defined by the intrinsic dynamics) toward the required pattern (Schöner & Kelso, 1988b). In this explicit sense, *information is meaningful and specific to the biological system only to the extent that it contributes to the order parameter dynamics attracting the system to the required (e.g., perceived, learned, memorized, intended) behavioral pattern*. An important consequence of this definition is that information is defined in the same space as the collective variables that characterize the pattern. In fact, this operational definition of information has no meaning whatsoever outside its influence on a set of collective variables and their intrinsic dynamics. The crux of this formulation is that information is arbitrary with respect to the dynamics it modifies.

This statement is not merely a claim or a philosophical comment (Kelso, Holt, Kugler, & Turvey, 1980; Kugler, Kelso, & Turvey, 1980; 1982) but rather can be shown to work (in the sense of an explicit mapping between experiment and theory) in a number

cases, including the formulation of perception-action patterns (Kelso, Delcolle, & Schöner, in press), the modification of action patterns by the environment (experiments by Tuller & Kelso, 1985, in press; see Schöner & Kelso, 1988c, 1988d for theory), and learned and memorized behavioral patterns (Yamanishi, Kawato, & Suzuki, 1980; Schöner & Kelso, 1988b, 1988c, 1988d). Many other experimental systems are open to a similar theoretical analysis: locomotory gait patterns and gait changes (Schöner, Jiang, & Kelso, 1988), the coordination of rhythmic movement between two human subjects (Schmidt, Carello, & Turvey, 1989), the mode-lockings studied by Kelso and DeGuzman (1988), and interlimb coordination of discrete movements (Kelso, Southard, & Goodman, 1979).

Let us fix the concept of behavioral information in a particular context that we have not discussed so far, namely, the ability of biological systems to change patterns of behavior flexibly in a purposeful or intentional fashion. Our previous empirical and theoretical work on spontaneous switching of coordination patterns leads to two general predictions. First, the intrinsic dynamics should influence the process of intentional change among available behavioral patterns. That is, the time it takes to switch from one pattern to the other depends on the stability of the patterns themselves. For example, if it is true that the anti-phase coordination pattern is less stable than the in-phase pattern, the system should switch faster in that direction than vice versa. The second prediction is that an intention, defined now as an intended behavioral pattern (and thus included as part of the behavioral dynamics) can change the dynamical characteristics, such as the stability, of the behavioral patterns. Thus, intentional information can be viewed as a perturbation of the intrinsic dynamics, attracting the system to the intended behavioral pattern. Note that the conceptual distinction between the intrinsic dynamics and the contribution of intention is meaningful only if the intrinsic dynamics are identified by experiments that do not involve intentional behavioral change. It is this essential feature of the approach that makes predictions about the process of behavioral change possible in the first place.

An easy way to see the consequences of these predictions is through the potential pictures shown in Figure 1.5. On the top left we show the standard intrinsic dynamics (Equation 3) of Haken et al. (1985) with two minima of the collective variable at  $\phi = 0^\circ$  and  $\phi = 180^\circ$ . On the top right we show the potentials corresponding to an intentional perturbation,

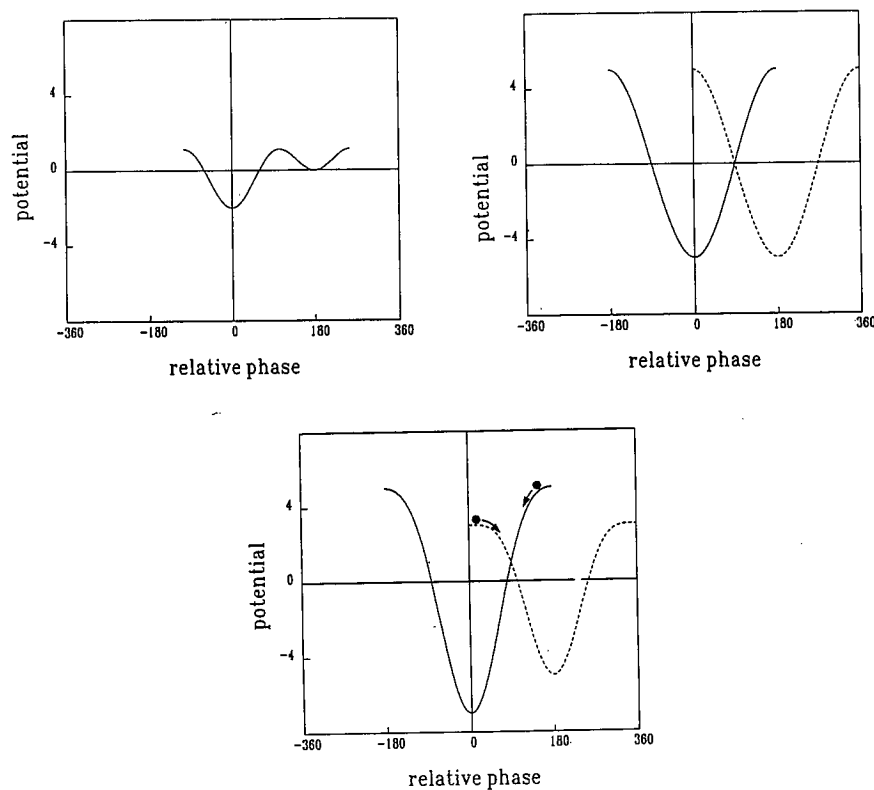


Figure 1.5. Modification of the intrinsic dynamics by behavioral information, in this case, an intended behavioral pattern. Upper left depicts the intrinsic dynamics according to the potential (3). Upper right shows the potential (9) specifying the intentional perturbation. Lower graph is the result of summing the top two to arrive at the full dynamics. The little ball travels much faster along the steeper slope of  $\phi = 0$  than along the slope of  $\phi = 180$ , consistent with empirical switching time data of Kelso et al. (1988).

$$V_{\psi}(\phi) = -c_{\text{intent}} \cos(\phi - \psi) \quad (9)$$

where the solid line represents an intentional pattern that is in-phase ( $\psi = 0$ ) and the dotted line an intentional pattern that is anti-phase ( $\psi = \pi$ ). A single parameter,  $c_{\text{intent}}$ , determines the strength of

the intention to produce one pattern or the other. Note that in this figure the intrinsic dynamics are absent, so the two patterns are equally stable.

The full dynamics, that is,  $V(\phi) + V(\phi)_{\text{intent}}$ , is shown bottom center. Now it is easy to visualize the consequences of the theory applied to intentional behavior (see Schöner & Kelso, 1988d, 1988e, for details). First, it is obvious that an intention can change the intrinsic dynamics; that is, it can destabilize one pattern and stabilize the other. Second, because the attractor at  $\phi = 0^\circ$  is more stable than at  $\phi = 180^\circ$ , the system will run faster to the in-phase pattern than to the anti-phase pattern. We have illustrated this situation with black balls in Figure 1.5. A recent experiment nicely demonstrated the influence of the intrinsic dynamics on intentional switching (Kelso, Scholz, & Schöner, 1988; Scholz & Kelso, in press-a). The task was initially to cycle the fingers either in-phase or anti-phase. Subjects were paced for 10 cycles by a metronome, which was then turned off. Instructions were to continue cycling at the initial frequency until an auditory tone signaled a switch to the opposite mode of coordination. The results, measured as mean switching time, show that switching from in-phase to anti-phase is about twice as slow as in the opposite direction, further evidence that the anti-phase mode is intrinsically less stable than the in-phase state.

Two points, one methodological and the other theoretical, emerge from these results. The first is that switching time, the tool employed here, reveals dynamic constraints on intentional switching that may be put to more general use. For instance, one may use the method "backwards" to identify the relative stabilities of behavioral patterns in those patterns for which it is difficult, if not impossible, to find phase transitions. The second point is that two languages that are often considered irreconcilable, namely, the language of intentionality and the language of dynamics, are actually captured in one unified picture. In the bimanual case, an intention acts in the same space of collective variables as that in which the intrinsic patterns are measured. Intentional information defines an attractor in that space and is meaningful to the extent that it attracts the system towards an intended behavioral pattern. At the same time, intentions are restricted by the intrinsic dynamics, in that the ability to perform a particular pattern is influenced by the relative stability of the available patterns. In short, intentions parameterize the dynamics but are in turn constrained by the dynamics.

Q. You have told me mostly about rhythmic movement of two components. What about other systems, other levels of description?

A. Earlier, we mentioned that speech research may benefit from the strategy we have formulated, particularly because this discipline has been searching for its own relevant variables for some time. The acoustic instantiation accompanying the production of a word varies with many factors, including stress pattern, the rate at which the word is produced, and the nature of the surrounding speech segments. However, this variation is not reflected in our perception of a spoken word. We seem able to decipher the linguistic content of an utterance despite the context-dependent nature of its production. Previously it was thought that a one-to-one relationship might exist between the electromyographic activity of individual muscles and the corresponding speech segments produced (Liberman, Cooper, Shankweiler, & Studdert-Kennedy, 1967). However, such activity was found to be highly context-dependent (MacNeilage & DeClerk, 1969). This finding led researchers to study the articulatory gestures associated with the speech segments (Liberman et al., 1967; Lindblom, 1963; Kozhevnikov & Chistovich, 1965) to determine whether the robust nature of speech perception lies in the articulatory movements themselves. These movements, however, displayed the same lack of invariance found in electromyographic activity. Presently, the conceptual basis for invariance is being challenged because this invariance means no change in the face of numerous metrical changes such as stress and speaking rate. But the noisiness of biological systems makes it difficult to characterize invariance as anything more than a statistical effect (see also Abbs & Connor, this volume).

Just as it has proven useful in the bimanual experiments, relative timing may be conceptually better suited to understanding of the speech system. This most recent tack (e.g., Kent & Netsell, 1971; Lofqvist & Yoshioka, 1981) has shown that relationships between articulators vary less across metric change than do absolute measures. This tack has also enabled investigators (Kelso, Saltzman, & Tuller, 1986) to introduce *stability* as a means of characterizing patterns in articulatory gestures. We stress, however, that this is not merely an exchange of terms. The advantage that accompanies stability is that fluctuations are no longer seen as a confound but as essential to characterizing the change from one pattern (phonemic utterance) to another.

An example of this insight applied to dynamic pattern theory is a study by Tuller and Kelso (in press), who investigated the relative timing between peak glottal opening and minimum lip aperture in two utterance conditions. Subjects were instructed to repetitively produce either a consonant-vowel (/pi/) or a vowel-consonant (/ip/) utterance to the beat of a visual metronome whose frequency was scaled similarly to that in previous bimanual experiments in our lab. The results displayed a consistent relative phase value for /pi/ across all frequencies. However, although /ip/ displayed a distinctly different value for relative phase than /pi/ did at lower frequencies, a transition to the relative phase corresponding to /pi/ ensued at higher frequencies. Thus, we have empirical evidence that the order parameter dynamics of relative phase is not unique to the bimanual system, but is useful in characterizing pattern stability and change in a completely different context. Furthermore, in a followup experiment using tape recordings of the actual trials from the production experiment, listeners were found to perceive the shifts in relative phase as corresponding shifts from /ip/ to /pi/ precisely at the point at which this transition occurred in the articulatory data. This remarkable fit between the production measures and perception of syllable form suggests that the relative phase dynamics may provide a means for understanding the coupling between speech production and perception.

A further step may be taken to link dynamic pattern theory to a completely different level of observation. Although the idea has not been explicitly or systematically tested to date, examples in the neurobiological literature have suggested that certain experimental aspects of neuronal behavior may be realizable within a dynamic pattern framework (Kelso et al., 1987; Schöner & Kelso, 1988a). In fact, typical phenomena of temporal ordering are widespread in the neuronal pattern generation literature, for example, synchronization, frequency locking, and phase locking (Rand, Cohen, & Holmes, 1988). These phenomena are typically depicted through measures of relative phase and latency among components, neuronal burst frequency, and frequency differences among neurons (e.g., Croll, Davis, & Kovac, 1985). Such findings strongly suggest that collective variables for temporal order at the neuronal level can be defined. Furthermore, observation at this level has uncovered numerous candidates for control parameters such as serotonin, whose concentration changes the elicitation threshold for rhythmic feeding patterns in *Helisoma* (see, e.g., Selverston & Moulins, 1987) or electrical stimulation, which has proven to



modify locomotory patterns in decerebrate cats (Shik, Severin, & Orlovskii, 1966) and fleeing behavior in hens (von Holst & von Saint Paul, 1973). Finally, we note that stability measures, although not explicitly formalized in the neurobiological literature, are evident. For example, fluctuation measures (Wyman, 1965) have been used in the analysis of flight patterns in the blowfly at the individual neuronal level, although collective variables were not identified and thus stability was not concretely ascertained. Nevertheless, once collective variables for neuronal patterns have been identified, the relaxation time and fluctuation measures are well defined and can be calculated through perturbation techniques and fluctuational analysis. These theoretical concepts and empirical techniques are the key to understanding pattern stability and change at the neuronal level.

*Q. Your discussion of pattern switching leads me to inquire about pattern selection. How, in this picture, does one particular pattern emerge from those available?*

A. Processes of pattern formation and selection occur throughout nature. Typical nonequilibrium systems will have many possible configurations of pattern available, not all of which are stable. A popular image is of multiple attractors with many basins of temporary attraction (Gleick, 1987). One of the challenges in nonlinear science is to understand the dynamics of sequencing among multiple patterns, namely, which patterns are explored and eventually selected. Again, such questions are relevant at multiple scales of observation and to multiple functions, from learning to the immune system (see contributions in, e.g., Koslow et al., 1987). Here are a few ways to think about pattern selection in the paradigm of nonlinear science. The list is far from inclusive, of course, and is restricted to issues germane to this book on motor coordination.

1. In our example of intentional switching among behavioral patterns, we saw that an "intention" can select a pattern, but which pattern can be most easily selected is determined by the pattern's stability. Selection, in other words, is influenced by the (quantitatively measurable) relative stability of the patterns available to the system. Our focus has been to identify *constraints* on the pattern selection rather than the dynamics of the selection process itself. For example, other factors (e.g., selection on the basis of cost or other performance criteria) are not accounted for in the behavioral dynamics. Such accounting would require degrees of freedom

in addition to those included in the intrinsic dynamics (cf. Schöner & Kelso, 1988e).

In the language of dynamic pattern theory, boundary conditions play a parametric role in pattern selection. Such parametric influences can be unspecific or specific. Generally, in nonequilibrium systems, boundary conditions may cause the system to "pin" itself to the most stable modal configuration (for examples see Campbell, 1987; Haken, 1983). The shape of the environment, for example, in Bénard convection favors the formation of certain patterns; that is, whether the enclosure is circular or rectangular will result in quite different patterns under the same values of the temperature gradient. Instabilities can be viewed as playing a role in pattern selection in the sense that these instabilities seek out the most stable pattern. Fluctuations are crucial because they probe the environment of the collective state, "selecting" (or leading to the emergence of) a new pattern.

In a number of experimental situations, for example, the perception-action patterns studied by Tuller and Kelso (in press) and the mode-locking studies of Kelso and DeGuzman (1988), the pattern that emerges (or is selected) is a direct consequence of *cooperative* and *competitive* interactions between the intrinsic dynamics and external influences. In the Tuller and Kelso data, for example, when an environmentally required relative phase coincides with one of the basic intrinsic patterns,  $\phi = 0$  or  $\phi = \pi$ , the minimum of the potential is exactly at the required relative phase, and its shape is well articulated (i.e., variation in  $\phi$  is small). There is, in other words, a *cooperation* between extrinsic (environmentally defined) and intrinsic dynamics. In contrast, if the environmentally required relative phase does not correspond to one of the intrinsic patterns, a *competition* ensues, pulling the minimum away from the required relative phase (i.e., variation in  $\phi$  is large). In short, the balance between intrinsic and extrinsic dynamics can be seen as a cooperative/competitive one. These processes thus determine (or select) which pattern is observed.

As a general point, nonlinear dynamical systems—even deterministic equations of motion—are enormously sensitive to initial conditions and parameters in certain regions of parameter space. Very complex behavior can emerge in systems governed by simple rules (e.g., the logistic equation [see May, 1976] or the circle map [see, e.g., Kelso & DeGuzman, 1988]). Until we understand these

systems better, we should probably resist ascribing a priori the process of pattern selection to an agency residing inside the system.

## CONCLUSION

We have presented, in a dialogue, some of the key features of the dynamic pattern strategy, which is aimed at understanding the coordination of behavior, its stability and change. The language of dynamic patterns stresses a close relation between theory and experiment and is level-independent in the sense that the central ideas and observables are applicable on several, potentially multiple, levels of observation. Key concepts are the characterization of behavioral patterns by collective variables, the determination of the dynamics of these patterns, and the study of their stability (and lack thereof). As Gould (1988) has recently remarked, "To know the reasons for infrequent change, one must understand the ordinary rules of stability" (p. 23). But the stability of what? The paradox, from our perspective, is that to understand stability, one must understand how it is lost because, as we have shown, loss of stability is central to behavioral change.

Identifying collective variables (order parameters in the language of synergetics) proves to be crucial to defining those aspects of behavior that are modifiable. Information with meaning is expressed in dynamic pattern language in terms of the same set of collective variables by which the behavioral patterns are characterized. This is a step toward the reduction, if not the elimination, of a traditional demarcation between mental (linguistic) and physical (dynamical) modes of description (cf. Pattee, 1976). When such information is included as part of the dynamics, the match between mathematically formulated theoretical predictions and empirical results is quite adequate.

The strategy of dynamic pattern theory offers a recipe, but the scientist must provide his or her own ingredients. It still demands insight and knowledge, particularly from the experimentalist, who must know the system sufficiently well to define the parameters that promote nonlinear behavior. It is a nontrivial step to find a qualitative change of behavior and to identify the conditions under which such change will emerge. Phase transitions are nevertheless crucial to understanding the dynamics that underlie both stability and change of coordinated behavior. They provide the physical

foundation upon which to build a deeper understanding of those essential biological and psychological functions that we all care about.

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